Test Booklet No:

DO NOT OPEN THE SEAL OF THE BOOKLET UNTIL YOU ARE TOLD TO DO SO

DIBRUGARH UNIVERSITY

ENTRANCE TEST FOR ADMISSION INTO THE MASTER OF SCIENCE IN MATHEMATICS, 2017

Time: 2 Hours **Maximum Marks: 100**

Read the instructions carefully before you begin to answer the questions. This Booklet contains questions in English only.

Before you start to answer the questions you must check up this Booklet and ensure that it contains all the pages (1 - 8) and see that no page is missing or repeated. If you find any defect in this Booklet, you must get it replaced immediately.

INSTRUCTIONS TO CANDIDATE

1. Fill in the particulars on this page of the Test Booklet with Blue / Black Ball Point pen.
2. This Booklet contains 50 questions in all each carrying 2 marks.
3. All questions are compulsory.
4. Each questions is followed by 4 (four) suggested answers, out of which only 1 (one) is correct. Correct answer must be shown by completely blackening the corresponding circles on Answer Sheet against the relevant question number by Blue / Black pen only.
5. There is negative marking for wrong answer at the rate of 25%.
6. Replacing the original answer by another will be treated as wrong answer.
7. Do not write anything on the backside of Response Sheet.
8. At the end of the Booklet, extra pages are provided for rough work and Mathematical calculations, if needed.
9. At the end of the examination, this booklet must be returned to the invigilator(s) before leaving the Examination Hall.
10. "Mobile phones and wireless communication devices are completely banned in the examination halls. Candidates are advised not to keep Mobile Phones / any other wireless communication devices with them even in switch off mode, in their own interest."

Roll Number :

Roll Number in words :

Invigilator's Signature : ______________________________

Instruction: Following questions are of multiple choice types. **Each question contains four (4) suggested options out of which only one (1) is correct. Each question carries two (2) marks for correct answer and** –0.5 **for each wrong attempt.** There will be no negative mark for the unanswered questions.

(1) Let M= $\begin{pmatrix} 1/2 & 1/4 \\ 0 & 1 \end{pmatrix}$ and x = $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$ then the value of $\lim_{n\to\infty} M^n x$ is

(A) Does not exist (B) $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ (C) $\begin{pmatrix} 2 \\ 4 \end{pmatrix}$ (D) $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$

(2) Let f : R →R be a differentiable function such that f(2) = 2, and

$|f(x) - f(y)| \le 5(|x - y|)^{3/2}$ $\forall$ x, y $\in$ R. Let $g(x) = x^3 f(x)$. Then $g'(2)$ is equal to

(A) 5 (B) 15/2 (C) 12 (D) 24

(3) Let $\{S_n\}_{n\ge2}$ be a sequence defined as $S_{n+1} = \frac{7S_n+11}{21}$, $S_1 = 1$. Which of the following statements is TRUE?

(A) It is an increasing sequence which diverges.

(B) It is an increasing sequence that converges to 11/14.

(C) It is a decreasing sequence which diverges.

(D) It is a decreasing sequence that converges to 11/14.

(4) Which of the following must be true of a continuous function on (a, b)?

(A) If f be a function such that f(a) = 2, f(b) = 5 then f(c) can be equal to 3 for some c $\notin$ (a, b).

(B) The function achieves its maximum on (a, b)

(C) The function is bounded

(D) The function must be differentiable on (a, b).

(5) Let p(x) be a real polynomial in x of degree 5, then the value of $\lim_{n\to\infty} \frac{p(n)}{2^n}$ is

(A) 5 (B) 0 (C) 1 (D) ∞

(6) p and q are two distinct odd primes such that $(p-1)\,|\,(q-1)$. If $\gcd(a, pq) = 1$ then

(A) $a^{p-1} \equiv 1(mod\ p)$

(B) $a^{q-1} \equiv 1(mod\ pq)$

(C) $p^{q-1} + q^{p-1} \equiv 1(mod\ pq)$

(D) $a^{pq} \equiv a(mod\ pq)$

(7) Let D be non-zero $n \times n$ real matrix with $n \geq 2$. Which of the following implication is valid?

(A) det(D) = 0 implies rank(D) = 0

(B) det(D) = 1 implies rank(D) $\neq$ 1

(C) rank(D) = 1 implies det(D) $\neq$ 1

(D) rank(D) = n implies det(D) = 0.

(8) Let A, B be $n \times n$ matrices. Which of the following equals *trace*(A^2B^2) ?

(A) $(\text{trace}(A^2B^2))^2$

(B) $\text{trace}(AB^2A)$

(C) $\text{trace}((AB)^2)$

(D) trace(BABA)

(9) Let $f : \mathbb{R} \to \mathbb{R}$ be a continuously differentiable function, with $f(0) = f(1) = f'(0) = 1$. Then

(A) f'' is the zero function

(B) $f''(0) = 0$

(C) $f''(x) = 0$ for some $x \in (0, 1)$

(D) f'' never vanishes.

(10) The number of element in the set $\{m : 1 \leq m \leq 1000,\ (m, 1000) = 1\}$ is

(A) 100 (B) 250 (C) 300 (D) 400

(11) Let $\left\{0, \frac{1}{2}, 1\right\}$ be three distinct points on [0, 1]. Let p be the unique interpolating polynomial of suitable degree on [0, 1] such that $p(0) = 0,\ p\left(\frac{1}{2}\right) = 0,\ p(1) = 1$, then $p\left(\frac{1}{4}\right)$ is equal to

(A) $-\frac{1}{8}$ (B) $-\frac{1}{2}$ (C) $\frac{2}{5}$ (D) $-\frac{1}{5}$

(12) In the solution of the following set of linear equations by Gauss elimination use partial pivoting $5x + y + 2z = 34,\ 4y - 3z = 12,\ 10x - 2y + z = -4$. The pivots for elimination of x and y are

(A) 10, 2 (B) 5, 4 (C) 5, −4 (D) 10, 4

(13) Consider the series $x_{n+1} = \frac{x_n}{2} + \frac{9}{8x_n}$, $x_0 = 0.5$ obtained from the Newton-Raphson method. The series converges to

(A) $\sqrt{2}$ (B) 1.6 (C) 1.5 (D) 1.4

(14) Given the equation $x^4 - x - 10 = 0$. The best possible choice for the equation is:

(A) the root of the given equation lies between 1 and 2.

(B) the root of the equation lies between 0 and 1.

(C) no root exists.

(D) the root of the equation is 1.855.

(15) Given

X	–4	–1	0	2	5
f(x)	1245	33	5	9	1335

The Newton's divided difference formula is

(A) $f(x) = 3x^4 + 5x^3 + 6x^2 - 14x + 5$ (B) $f(x) = \sin x \cos x + 6x^2 + 6$

(C) $f(x) = 3x^4 - 5x^3 + 6x^2 - 14x + 5$ (D) $f(x) = \sin x \cos x - 6x^2 + 6$

(16) A feasible solution to a general L.P.P. which is also a basic solution is called a

(A) feasible solution (B) basic solution

(C) basic feasible solution (D) Non of these

(17) Which of the following changes the nature of the constraints in the given L.P.P.

(A) Artificial variables (B) Slack variables

(C) Surplus variables (D) None of these

(18) The number of basic solution to the system

$x_1 + 2x_2 + 10x_3 + 4x_4 = 5$

$x_1 + x_2 + 5x_3 + 4x_4 = -8$

is

(A) 0 (B) 4 (C) 6 (D) Infinite

(19) Which of the following set is a convex set ?

(A) $\{(x,y): x^2+y^2 \geq 1\}$ (B) $\{(x,y): y^2 \geq x\}$

(C) $\{(x,y): y \geq 2, y \leq 4\}$ (D) $\{(x,y): 3x^2+3y^2 \geq 5\}$

(20) The L.P.P

$Max\ z = 3x_1 - 4x_2$

subject to

$x_1 - x_2 \geq 0$

$x_1 \leq 6$

$x_1, x_2 \geq 0$

has

(A) no solution (B) multiple optimal solution

(C) unique optimal solution (D) unbounded solution

(21) Let Z be the group of all integers under the operation of addition. Which of the following subsets of G is not a subgroup of Z?

(A) $\{0\}$

(B) $\{n \in Z | n \geq 0\}$

(C) $\{n \in Z |\ n$ is an even integer$\}$

(D) $\{n \in Z | n$ is divisible by both 6 and 9$\}$

(22) A cyclic group of order 15 has an element xx such that the set $\{x^3, x^5, x^9\}$ has exactly two elements. The number of elements in the set $\{x^{13n} | n$ is a positive integer$\}$ is

(A) 3 (B) 4 (C) 8 (D) infinite

(23) If S is a ring with the property that $s = s^2$, for each $s \in S$, which of the following must be true ?

(I) $s + s = 0$, for each $s \in S$.

(II) $(s+t)^2 = s^2 + t^2$, for each $s, t \in S$.

(III) S is commutative.

(A) III only (B) I and III only (C) II and III only (D) I, II and III

(24) Up to isomorphism, how many additive abelian groups G of order 16 have the property that $x+x+x+x=0$ for each x in G?

(A) 0 (B) 1 (C) 2 (D) 3

(25) Let R be a ring with a multiplicative identity. If U is an additive subgroup of R such that $ur \in U$ for all $u \in U$ and for all $r \in R$, then U is said to be a right ideal of R. If R has exactly two right ideals, which of the following must be true?

I. R is commutative.

II. R is a division ring (that is, all elements except the additive identity have multiplicative inverses).

III. R is infinite.

(A) I only (B) II only (C) III only (D) I, II, and III

(26) If b and c are elements of a group G, and if $b^5 = c^3 = e$ where e is the unit element of G then the inverse of $b^2cb^4c^2$ must be

(A) b^3c^2bc (B) $b^4c^2b^2c$ (C) $c^2b^4cb^2$ (D) cbc^2b^3

(27) Let R denotes the field of real numbers, Q the field of rational numbers, and Z the ring of integers. Which of the following subsets F_i of R, $1 \le i \le 4$, are subfield of R?

$F_1 = \{a/b | a, b \in Z \text{ and } b \text{ is odd}\}$; $F_2 = \{a + b\sqrt{2} | a, b \in Q\}$;

$F_3 = \{a + b\sqrt{2} | a, b \in R\}$; $F_4 = \{a + b\sqrt[4]{2} | a, b \in Q\}$

(A) No F_i is a subfield of R (B) F_3 only (c) F_2 and F_3 only.

(d) F_1, F_2 and F_3 only.

(28) Let G_n denote the cyclic group of order n. which of the following direct product is not cyclic?

(A) $G_{17} \times G_{11}$ (B) $G_{17} \times G_{11} \times G_5$ (C) $G_{17} \times G_{33}$ (D) $G_{22} \times G_{33}$

(29) Which of the following subsets are subrings of the ring of real numbers?

I. $\{a + b\sqrt{2} : \text{a and b are rational}\}$

II. $\left\{\frac{n}{3^m} : n \text{ is an integer and m is a non-negativeinteger}\right\}$

III. $\{a + b\sqrt{5} : \text{a and b are realnumbers and } a^2 + b^2 \le 1\}$

(A) I only (B) I and II only (C) I and III only (D) III only

(30) Jacobi's method is also known as

(A) Displacement method (B) Simultaneous displacement method

(C) Simultaneous method (D) Diagonal method

(31) The convergence of which of the following method is sensitive to starting value?

(A) False position (B) Gauss Seidel method

(C) Newton-Raphson method (D) All of these

(32) Let $f:[2,4]\to R$ be a continuous function such that $f(2)=3$ and $f(4)=6$. The most we can say about the set $f([2,4])$ is that

(A) It is a set which contains $[3,6]$.
(B) It is a closed interval.
(C) It is a set which contains 3 and 6.
(D) It is a closed interval which contains $[3,6]$.

(33) Suppose that $f:Q\to R$ is a continuous function where Q is a set of rational numbers. Then

(A) f is unbounded.
(B) f is differentiable.
(C) f is not differentiable.
(D) f is unbounded and differentiable.

(34) The order and degree of the following differential equation

$$3t^2\left(\frac{dy}{dt}\right)^3-(\sin t)\left(\frac{d^2y}{dt^2}\right)^6=0 \text{ is}$$

(A) 6 & 2 (B) 2& 6 (C) 2& 3 (D) None of the above

(35) Solution of the differential equation $ydy+(y^2+1)dx=0$ is

(A) $y^2=-1+e^{2c-2x}$ (B) $y^2=1+e^{2c-2x}$

(C) $y=-1+e^{2c-2x}$ (D) None of the above

(36) Solution of the differential equation $dy+(y^2+1)dx=0$ is

(A) $y=tan^{-1}x+c$ (B) $y=\tan(x+c)$ (C) $y=0$ (D) None of the above

(37) The differential equation $(3x^4y^2 - x^2)dy + (4x^3y^3 - 2xy)dx = 0$ is

(A) Exact (B) not exact (C) may not be exact (D) none of the above

(38) Choose the correct one:

(A) $div(ab) = a.div\ b + b\ grad\ a$
(B) $div(ab) = a\ div\ b + b\ grad\ \ a$
(C) $div(ab) = a\ div\ b + b.grad\ a$
(D) None of the above

(39) A vector whose divergence is zero, is called

(A) Irrotational vector
(B) Solenoidal vector
(C) Polar vector
(D) None of the above

(40) Two surfaces $\phi(x, y, z) = a$ & $\psi(x, y, z) = b$ will cut orthogonally, if

(A) $\nabla\phi.\nabla\psi = 0$
(B) $\nabla^2\phi = 0, \nabla^2\psi = 0$
(C) $\nabla^2\phi \neq 0, \nabla^2\psi \neq 0$
(D) None of the above

(41) The angle between the lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{1}$ & $\frac{x}{1} = \frac{y}{-1} = \frac{z}{1}$ is

(A) 0 degree
(B) 45 degree
(C) 90 degree
(D) None of the above

(42) The equation of the straight line through (2, 1, -2) and equally inclined to the axes is

(A) $x - 2 = y + 1 = z - 2$ (B) $x + 2 = y + 1 = z - 2$

(C) $x - 2 = y - 1 = z - 2$ (D) None of the above

(43) The equation of the sphere passes through the origin and has its centre at $(\frac{1}{2}, \frac{1}{2}, 0)$ is
(A) $2x^2 + 2y^2 + 2z^2 = 1$
(B) $x^2 + y^2 + z^2 = \frac{1}{4}$
(C) $2x^2 + 2y^2 + z^2 = \frac{1}{2}$
(D) None of the above

(44) A body of 65 kg force is suspended by two strings of lengths 5 and 12 meters attached to two points in the same horizontal line whose distance apart is 12 meters. The tension of strings are

(A) 60 & 25 (B) 45 & 25 (C) 45 & 60 (D) 20 & 25

(45) If a system of forces acting on a body is in equilibrium and the body undergo a slight displacement consistent with the geometrical conditions of the system, the algebraic sum of virtual works is

(A) Always non-zero (B) zero (C) infinite (D) Finite but may or may not be zero

(46) A system of forces, acting in one plane upon a rigid body, is in equilibrium, if

(A) The sum of their components parallel to each of two lines in their plane is zero

(B) The algebraic sum of their moments about any point is zero

(C) Both a and b.

(D) Neither a nor b.

(47) Let f be an entire function. If $\operatorname{Re} f$ is bounded then

(A) $\operatorname{Im} f$ is constant (B) f is constant

(C) $f \equiv 0$ (D) f' is non zero constant

(48) Find the correct statement.

(A) The point $3+4i$ lies inside the circle $|z-i|=4$

(B) The point $3+4i$ lies outside the circle $|z-i|=4$

(C) The point $3+4i$ lies on the circle $|z-i|=4$

(D) The point $3+4i$ lies on the centre of the circle $|z-i|=4$

(49) Which of the following statements is wrong?

(A) $|z| \ge |\operatorname{Re} z| + |\operatorname{Im} z|$ (B) $2|z| \ge |\operatorname{Re} z| + |\operatorname{Im} z|$

(C) $|z| \ge |\operatorname{Im} z|$ (D) $|z| \ge |\operatorname{Re} z|$

(50) Suppose that $f(z)$ is analytic function inside and on a simple closed curve C. Then

(A) $\int_C f(z) > 0$ (B) $\int_C f(z) < 0$

(C) $\int_C f(z) = 0$ (D) $\int_C f(z)$ is a non-zero complex number.

Answer Sheet

Roll No: | | | | |

Invigilator's Signature: ____

1.	Ⓐ Ⓑ Ⓒ Ⓓ	26.	Ⓐ Ⓑ Ⓒ Ⓓ
2.	Ⓐ Ⓑ Ⓒ Ⓓ	27.	Ⓐ Ⓑ Ⓒ Ⓓ
3.	Ⓐ Ⓑ Ⓒ Ⓓ	28.	Ⓐ Ⓑ Ⓒ Ⓓ
4.	Ⓐ Ⓑ Ⓒ Ⓓ	29.	Ⓐ Ⓑ Ⓒ Ⓓ
5.	Ⓐ Ⓑ Ⓒ Ⓓ	30.	Ⓐ Ⓑ Ⓒ Ⓓ
6.	Ⓐ Ⓑ Ⓒ Ⓓ	31.	Ⓐ Ⓑ Ⓒ Ⓓ
7.	Ⓐ Ⓑ Ⓒ Ⓓ	32.	Ⓐ Ⓑ Ⓒ Ⓓ
8.	Ⓐ Ⓑ Ⓒ Ⓓ	33.	Ⓐ Ⓑ Ⓒ Ⓓ
9.	Ⓐ Ⓑ Ⓒ Ⓓ	34.	Ⓐ Ⓑ Ⓒ Ⓓ
10.	Ⓐ Ⓑ Ⓒ Ⓓ	35.	Ⓐ Ⓑ Ⓒ Ⓓ
11.	Ⓐ Ⓑ Ⓒ Ⓓ	36.	Ⓐ Ⓑ Ⓒ Ⓓ
12.	Ⓐ Ⓑ Ⓒ Ⓓ	37.	Ⓐ Ⓑ Ⓒ Ⓓ
13.	Ⓐ Ⓑ Ⓒ Ⓓ	38.	Ⓐ Ⓑ Ⓒ Ⓓ
14.	Ⓐ Ⓑ Ⓒ Ⓓ	39.	Ⓐ Ⓑ Ⓒ Ⓓ
15.	Ⓐ Ⓑ Ⓒ Ⓓ	40.	Ⓐ Ⓑ Ⓒ Ⓓ
16.	Ⓐ Ⓑ Ⓒ Ⓓ	41.	Ⓐ Ⓑ Ⓒ Ⓓ
17.	Ⓐ Ⓑ Ⓒ Ⓓ	42.	Ⓐ Ⓑ Ⓒ Ⓓ
18.	Ⓐ Ⓑ Ⓒ Ⓓ	43.	Ⓐ Ⓑ Ⓒ Ⓓ
19.	Ⓐ Ⓑ Ⓒ Ⓓ	44.	Ⓐ Ⓑ Ⓒ Ⓓ
20.	Ⓐ Ⓑ Ⓒ Ⓓ	45.	Ⓐ Ⓑ Ⓒ Ⓓ
21.	Ⓐ Ⓑ Ⓒ Ⓓ	46.	Ⓐ Ⓑ Ⓒ Ⓓ
22.	Ⓐ Ⓑ Ⓒ Ⓓ	47.	Ⓐ Ⓑ Ⓒ Ⓓ
23.	Ⓐ Ⓑ Ⓒ Ⓓ	48.	Ⓐ Ⓑ Ⓒ Ⓓ
24.	Ⓐ Ⓑ Ⓒ Ⓓ	49.	Ⓐ Ⓑ Ⓒ Ⓓ
25.	Ⓐ Ⓑ Ⓒ Ⓓ	50.	Ⓐ Ⓑ Ⓒ Ⓓ