

HIMACHAL PRADESH BOARD OF SCHOOL EDUCATION, DHARAMSHALA

CLASS: XII
SUBJECT : MATHEMATICS (Full Syllabus)

TIME ALLOWED: 3 HOURS

MAX.MARKS:80

Special Instructions:

- i. While answering your Questions, you must indicate on your answer-book the same Question No. as appear in your Questions Paper.
- ii. All Questions are compulsory.
- iii. Internal choices have been provided in some questions. Attempt only one of the choices in such questions.
- iv. Do not leave blank page / pages in your answer book.
- v. Question numbers 1 – 16 are multiple choice questions (M.C.Q.) carrying 1 mark each.
- vi. Question numbers 17 – 25 are of 3 marks each.
- vii. Question numbers 26 – 28 are of 4 marks each.
- viii. Question numbers 29 – 33 are of 5 marks each.
- ix. Graph paper must be attached in between the answer book.

Q.1. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined as $f(x) = 3x$ (1)
Choose the correct answer:

- (a) f is one-one onto (b) f is many one onto
(c) f is one-one but not onto (d) f is neither one-one nor onto

Q.2. $\tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3})$ is equal to (1)

- (a) $2\sqrt{2}$ (b) π (c) $-\frac{\pi}{2}$ (d) 0

Q.3. If $\sin^{-1}x = y$ then: (1)

- (a) $0 \leq y \leq \pi$ (b) $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$
(c) $0 < y < \pi$ (d) $-\frac{\pi}{2} < y < \frac{\pi}{2}$

Q.4. The number of all possible matrices of order 3×3 with each entry 0 or 1 is: (1)

- (a) 27 (b) 18 (c) 81 (d) 512

Q.5. The second order derivative of $\log x$ is: (1)

- (a) $\frac{1}{x}$ (b) $\frac{1}{x^2}$ (c) $-\frac{1}{x^2}$ (d) None of these

Q.6. The rate of change of the area of the circle with respect to its Radius $r = 6$ cm is: (1)

- (a) 10π cm (b) 12π cm (c) 8π cm (d) 11π cm

Q.7. The approximate change in the volume of a cube of side x meters caused by increasing the side by 3 % is: (1)

- (a) $0.09 x^3 m^3$ (b) $0.9 x^3 m^3$ (c) $0.06 x^3 m^3$ (d) $0.6 x^3 m^3$

Q.8. $\int e^{x \sin^{-1} t} (1 + t \cos^{-1} t) dt$ equals to: (1)

- (a) $e^{x \sin^{-1} t} + C$ (b) $e^{x \cos^{-1} t} + C$ (c) $e^{x \sin^{-1} t} + C$ (d) $e^{x t \cos^{-1} t} + C$

Q.9. Area lying between the curves $y^2 = 4x$ and $y = 2x$ is: (1)

- (a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{3}{4}$

Q.10. The order of the differential equation (1)

$$2x^2 \frac{d^2 y}{dx^2} - 3 \frac{dy}{dx} + y = 0 \text{ is:}$$

- (a) 2 (b) 1 (c) 0 (d) not defined

Q.11. The integrating factor of the differential equation (1)

$$x \frac{dy}{dx} - y = 2x^2 \text{ is:}$$

- (a) e^{-x} (b) e^{-y} (c) x (d) $\frac{1}{x}$

Q.12. If $\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}|$, Then $\theta = ?$ (1)

- (a) $\frac{\pi}{4}$ (b) 0 (c) π (d) $\frac{\pi}{2}$

Q.13. Direction cosines of z- axis are: (1)

- (a) 0, 1, 0 (b) 1, 0, 0 (c) 0, 0, 1 (d) None of these

Q.14. Distance of the plane $2x - y + 2z + 3 = 0$ from the point (3, -2, 1) is: (1)

- (a) $\frac{3}{1}$ (b) $\frac{1}{3}$ (c) 0 (d) 13

Q.15. The probability of obtaining an even prime number on each die, when a pair of dice is rolled is: (1)

- (a) $\frac{1}{3}$ (b) $\frac{1}{3}$ (c) $\frac{1}{1}$ (d) 0

Q.16. If A and B are events such that $P(A/B) = P(B/A)$, Then: (1)

- (a) $A \subset B$ but $A \not\subset B$ (b) $A = B$ (c) $A \cap B = \emptyset$ (d) $P(A) = P(B)$

Q.17. Find $g \circ f$ and $f \circ g$, if (3)

$$f(x) = |x| \text{ and } g(x) = |5x - 2|$$

Q.18. Using elementary transformations, find the inverse of (3)

$$\begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$$

Q.19. Using the properties of the determinants, show that (3)

$$\begin{vmatrix} y+k & y & y \\ y & y+k & y \\ y & y & y+k \end{vmatrix} = k^2(3y+k)$$

Q.20. Find the value of k so that the function f defined by (3)

$$f(x) = \begin{cases} k + 1, & \text{if } x \leq \pi \\ \cos x, & \text{if } x > \pi \end{cases} \text{ is continuous at point } x = \pi$$

Q.21. Evaluate $\int_0^{\pi} x \ln 2x \, dx$ (3)

Q.22. By using the properties of the definite integrals, evaluate (3)

$$\int_0^{\frac{\pi}{2}} \frac{\cos^5 x}{\sin^5 x + \cos^5 x} \, dx$$

Q.23. Solve the differential equation: (3)

$$(x^2 - y^2) \, dx + 2xy \, dy = 0$$

OR

Solve the differential equation:

$$x \log x \frac{dy}{dx} + y = \frac{2}{x} \log x$$

Q.24. Two cards are drawn at random and without replacement from a pack of 52 playing cards. Find the probability that both the cards are black. (3)

Q.25. Find the probability distribution of number of tails in the simultaneous tosses of 3 coins. (3)

OR

Find the probability of getting 5 exactly twice in 7 throws of a die.

Q.26. Prove that (4)

$$\tan^{-1} \frac{2}{1} + \tan^{-1} \frac{7}{2} = \tan^{-1} \frac{1}{2}$$

OR

Express $\tan^{-1} \left(\frac{x}{a^2 - x^2} \right), |x| < a$ in the simplest form:

Q.27. Differentiate $\sin\{\tan^{-1}(e^x)\}$ with respect to x . (4)

OR

If $y^x = x^y$, find $\frac{d}{dx}$

Q.28. Find the area of a parallelogram whose adjacent sides are determined by the vectors. (4)

$$\vec{a} = i - j + 3k \text{ and } \vec{b} = 2i - 7j + k$$

Q.29. Solve the system of linear equations, using matrix method. (5)

$$\begin{aligned} x - y + z &= 4 \\ 2x + y - 3z &= 0 \\ x + y + z &= 2 \end{aligned}$$

Q.30. Find two positive numbers x and y such that $x + y = 60$ and xy^3 is maximum. (5)

OR

Find the equation of tangent and normal to the parabola

$$y^2 = 4ax \text{ at point } (at^2, 2at)$$

Q.31. Find the area of the region bounded by the ellipse (5)

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

OR

Using integration find the area of region bounded by the triangle whose vertices are $(-1,0)$, $(1,3)$ and $(3,2)$

Q.32. Find the shortest distance between the lines (5)

$$\vec{r} = (\lambda + 2\mu + \hat{k}) + (\hat{i} - \mu + \hat{k})$$

$$\vec{r} = (2\lambda - \mu - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$$

OR

Find the equation of plane through the intersection of the planes
 $3x - y + 2z - 4 = 0$ and $x + y + z - 2 = 0$ and passes through the point $(2, 2, 1)$.

Q.33. Solve the following linear programming problem (LPP) graphically:
 Maximize $Z = 5x + 3y$ subject to the constraints (5)

$$\begin{array}{rcl} 3x + 5y & = & 15 \\ 5x + 2y & = & 10 \\ x & \geq & 0 \\ y & \geq & 0 \end{array}$$