CBSE Sample Paper Class 11 Maths Solution

1. Solution:

We have,

$\sin(\sin^{-1}\frac{3}{5} + \cos^{-1} x) = 1$

$\Rightarrow \sin^{-1}\frac{3}{5} + \cos^{-1} x = \sin^{-1}(1)$ [$\because \sin\theta = x \Rightarrow \theta = \sin^{-1} x$]

$\Rightarrow \sin^{-1}\frac{3}{5} + \cos^{-1} x = \sin^{-1}(\sin\frac{\pi}{2})$ [$\because \sin\frac{\pi}{2} = 1$]

$\Rightarrow \sin^{-1}\frac{3}{5} + \cos^{-1} x = \frac{\pi}{2}$ ... (1)

We know that,

$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$, $x \in [-1,1]$ ...(2)

Equating equations (1) and (2), we get

LHS of both equations are equal.

$\sin^{-1}\frac{3}{5} + \cos^{-1} x = \sin^{-1} x + \cos^{-1} x$

$\Rightarrow \sin^{-1}\frac{3}{5} = \sin^{-1} x$

Hence, $x = \frac{3}{5}$.

2. Solution:

We have, $(3+7i)^2$

$(3+7i)^2 = 3^2 + (7i)^2 + 2(3)(7i)$

$= 9 + 49i^2 + 42i$

$= 9 - 49 + 42i$ [$\because i^2 = -1$]

$= -40 + 42i$

3. Solution:

Let $A = \begin{vmatrix} \cos 18^\circ & \sin 18^\circ \\ \sin 72^\circ & \cos 72^\circ \end{vmatrix}$

On expanding, we get

$A = (\cos 18^\circ \cdot \cos 72^\circ - \sin 18^\circ \cdot \sin 72^\circ)$

$= \cos(18^\circ + 72^\circ)$ $[\because \cos A \cdot \cos B - \sin A \cdot \sin B = \cos(A+B)]$

$= \cos 90^\circ = 0$ $[\because \cos 90^\circ = 0]$

OR

Solution:

We have, $\begin{vmatrix} x & 10 \\ 5 & 2x \end{vmatrix} = 0$

On expanding, we get

$\Rightarrow x \cdot 2x - 5 \cdot 10 = 0$

$\Rightarrow 2x^2 - 50 = 0$

$\Rightarrow 2x^2 = 50$

$\Rightarrow x^2 = 25$

$\Rightarrow x = \pm 5$

Hence, $x = \pm 5$.

4. Solution:

We have, $\lim_{x \to 1} \frac{x^{45} - 1}{x^{40} - 1}$

$= \lim_{x \to 1} \left\{ \left(\frac{x^{45} - 1}{x - 1} \right) \div \left(\frac{x^{40} - 1}{x - 1} \right) \right\}$

$= \lim_{x \to 1} \left(\frac{x^{45} - 1}{x - 1} \right) \div \lim_{x \to 1} \left(\frac{x^{40} - 1}{x - 1} \right)$

$= (45 \times 1^{44}) \div (40 \times 1^{39}) = \frac{45}{40} = \frac{9}{8}$

Hence, $\lim_{x \to 1} \frac{x^4 - 1}{x^5 - 1} = \frac{9}{8}$

4

0

SECTION B

5. Solution:

We have,

$\tan^{-1}\frac{1}{\sqrt{3}} + \cot^{-1} x = \frac{\pi}{2}$ ... (1)

We know that,

$\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}, \quad x \in R$ ...(2)

Equating equations (1) and (2), we get

$\tan^{-1}\frac{1}{\sqrt{3}} + \cot^{-1} x = \tan^{-1} x + \cot^{-1} x$

$\Rightarrow \tan^{-1}\frac{1}{\sqrt{3}} = \tan^{-1} x$

Hence, $x = \frac{1}{\sqrt{3}}$

6. Solution:

We have,

$$\sec\theta \begin{bmatrix} \sec\theta & \tan\theta \\ \tan\theta & \sec\theta \end{bmatrix} - \tan\theta \begin{bmatrix} \tan\theta & \sec\theta \\ \sec\theta & \tan\theta \end{bmatrix}$$

$$= \begin{bmatrix} \sec^2\theta & \sec\theta \cdot \tan\theta \\ \sec\theta \cdot \tan\theta & \sec^2\theta \end{bmatrix} - \begin{bmatrix} \tan^2\theta & \sec\theta \cdot \tan\theta \\ \sec\theta \cdot \tan\theta & \tan^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} \sec^2\theta - \tan^2\theta & \sec\theta \cdot \tan\theta - \sec\theta \cdot \tan\theta \\ \sec\theta \cdot \tan\theta - \sec\theta \cdot \tan\theta & \sec^2\theta - \tan^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} \sec^2\theta - \tan^2\theta & 0 \\ 0 & \sec^2\theta - \tan^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I = \text{Unit matrix} \quad [\because \sec^2\theta - \tan^2\theta = 1]$$

OR

Solution:

Since, $|A^3| = 512$

$$\Rightarrow |A^3| = 8^3$$

$\therefore A = \begin{bmatrix} \alpha & 1 \\ 1 & \alpha \end{bmatrix}$

$\Rightarrow \alpha^2 - 1 = 8$

$\Rightarrow \alpha^2 = 8 + 1$

$\Rightarrow \alpha^2 = 9$

$\Rightarrow \alpha = \pm 3$

Hence, $\alpha = \pm 3$.

7. Solution:

Here, $a = 32, d = 36 - 32 = 4$ and $a_n = 320$

Let n be the number of terms.

Now, $a_n = 320 \Leftrightarrow a + (n-1)d = 320$

$\Leftrightarrow 32 + (n-1)(4) = 320 \, [\because a = 32, d = 4]$

$\Leftrightarrow 32 + 4n - 4 = 320$

$\Leftrightarrow 4n + 28 = 320 \Leftrightarrow 4n = 320 - 28 = 292$

$\Leftrightarrow n = \frac{292}{4} = 73$

Hence, the given AP contains 73 terms.

OR

Solution:

We have,

$(n+2)(n+1)\times n! = 90\times n!$

$\Rightarrow (n+2)(n+1) = 90$

$\Rightarrow n^2+2n+n+2 = 90$

$\Rightarrow n^2+3n-88 = 0$

$\Rightarrow n^2+11n-8n-88 = 0$

$\Rightarrow n(n+11)-8(n+11) = 0$

$\Rightarrow (n+11)(n-8) = 0$

$\Rightarrow n = 8$

$[\therefore n\geq 0]$

Hence, $n = 8$.

8. Solution:

We have, $\frac{6+\sqrt{5}i}{1-\sqrt{5}i}$

$$\frac{6+\sqrt{5}i}{1-\sqrt{5}i} = \frac{6+\sqrt{5}i}{1-\sqrt{5}i}\times\frac{1+\sqrt{5}i}{1+\sqrt{5}i}$$

$$= \frac{6+6\sqrt{5}i+\sqrt{5}i+(\sqrt{5}i)^2}{1^2-(\sqrt{5}i)^2}\ [\because a^2-b^2=(a+b)(a-b)]$$

$$= \frac{6+7\sqrt{5}i+5i^2}{1-5i^2} = \frac{6+7\sqrt{5}i-5}{1+5}\ [\because i^2=-1]$$

$$= \frac{1+7\sqrt{5}i}{6} = \frac{1}{6}+\frac{7\sqrt{5}}{6}i$$

Hence, $\frac{6+\sqrt{5}i}{1-\sqrt{5}i} = \frac{1}{6}+\frac{7\sqrt{5}}{6}i$.

9. Solution:

Given, $f(x) = 256x^4$ and $g(x) = x^{\frac{1}{4}}$

$\therefore gof(x) = g[f(x)] = g(256x^4)$

$= (256x^4)^{\frac{1}{4}} = (256)^{\frac{1}{4}}\cdot(x^4)^{\frac{1}{4}}$

$= (4^4)^{\frac{1}{4}}\cdot(x^4)^{\frac{1}{4}} = 4^{4\times\frac{1}{4}}\cdot x^{4\times\frac{1}{4}} = 4x$

Hence, $gof(x) = 4x$.

OR

Solution:

Here, g is the inverse of $f(x)$.

$\Rightarrow fog(x) = x$

On differentiating with respect to x, we get

$f'\{g(x)\} \times g'(x) = 1$

$\Rightarrow g'(x) = \frac{1}{f'\{g(x)\}}$

$= \frac{1}{1+\{g(x)\}^{19}}$ $\left[\because f'(x) = \frac{1}{1+x^{19}}\right]$

$\Rightarrow g'(x) = 1 + \{g(x)\}^{19}$

Hence, the value of $g'(x)$ is $1 + \{g(x)\}^{19}$.

10. Solution:

The given equation of the line is $\frac{5-x}{7} = \frac{y}{4} = \frac{3-z}{4}$.

It can be rewritten in standard form

$\frac{x-5}{-7} = \frac{y}{4} = \frac{z-3}{-4}$.

1. Direction ratio of the line is $(-7, 4, -4)$.

Now, $\sqrt{(-7)^2 + (4)^2 + (-4)^2}$

$= \sqrt{49+16+16} = \sqrt{81} = 9$ Units

2. Direction cosine of the line is $\left(-\frac{7}{9}, \frac{4}{9}, -\frac{4}{9}\right)$

11. Solution:

We have, $(x^3+4y)^4$

Using the binomial theorem, we get

$$(x^3+4y)^4 = {}^4C_0 (x^3)^4 + {}^4C_1 (x^3)^3(4y) + {}^4C_2 (x^3)^2(4y)^2 + {}^4C_3 (x^3)^1(4y)^3 + {}^4C_4 (4y)^4$$

$$=1\cdot x^{12}+4\cdot x^9\cdot(4y)+6\cdot x^6\cdot 16\cdot y^2+4\cdot x^3\cdot 64\cdot y^3+1\cdot 256\cdot y^4$$

$$=x^{12}+16\cdot x^9\cdot y+96\cdot x^6\cdot y^2+256\cdot x^3\cdot y^3+256\cdot y^4$$

Hence, $(x^3+4y)^4=x^{12}+16x^9y+96x^6y^2+256x^3y^3+256y^4$

12. Solution:

We know that the equation of a line with slope m and $x-intercept$ d is given by:

$y=m(x-d)$.

Here, $\tan\theta=\frac{1}{5}$ and $d=6$.

Hence, the required equation of the line is:

$y=\frac{1}{5}(x-6)$

$\Rightarrow 5y=x-6$

$\Rightarrow x-5y-6=0$

13. Solution:

Clearly, $f: R\rightarrow R$ is a one - one function.

So, it is invertible.

Let $f(x)=y$.

Then, $11x-13=y \Rightarrow 11x=y+13$

$\therefore x=\frac{y+13}{11}$.

$\Rightarrow f^{-1}(y) = \frac{y+13}{11}$

Hence, $f^{-1}(x) = \frac{x+13}{11}$.

14. Solution:

The given function is:

$f(x) = \frac{6}{4}x^4 - 2x^3 - 6x^2 + 32$

On differentiating both sides $w.r.t.x$, we get

$f'(x) = \frac{6}{4} \cdot 4 \cdot x^3 - 2 \cdot 3 \cdot x^2 - 6 \cdot 2 \cdot x + 0$

$\Rightarrow f'(x) = 6x^3 - 6x^2 - 12x$

For strictly increasing or strictly decreasing,

Put $f'(x)=0$, we get

$6x^3 - 6x^2 - 12x = 0$

$\Rightarrow 6x(x^2 - x - 2) = 0$

$\Rightarrow 6x(x^2 - 2x + x - 2) = 0$

$\Rightarrow 6x(x+1)(x-2) = 0$

$\Rightarrow x = 0, -1 \; or \; 2$

Now, we find intervals in which $f(x)$ is strictly increasing or strictly decreasing.

Interval	$f'(x) = 6x(x+1)(x-2)$	Sign of $f'(x)$
$x < -1$	$(-)(-)(-)$	$-ve$
$-1 < x < 0$	$(-)(+)(-)$	$+ve$
$0 < x < 2$	$(+)(+)(-)$	$-ve$
$x > 2$	$(+)(+)(+)$	$+ve$

We know that, a function $f(x)$ is said to be strictly increasing, if $f'(x)>0$ and it is said to be strictly increasing, if $f'(x)<0$. So, the given function $f(x)$ is

1. Strictly increasing on the interval $(-1,0)$ and $(2,\infty)$.

2. Strictly decreasing on the interval $(-\infty,-1)$ and $(0,2)$.

15. Solution:

There are 5 letters in the word 'PUNAM', of which all are each of its own kind.

After fixing P at first place and M at last place, we have 3 letters out of which all are its own kind.

So, total number of words

$= 3! = 3\times2\times1 = 6$

After fixing P at first place, we have 4 letters out of which are all the each of its own kind

So, total number of words

$= 4! = 4\times3\times2\times1 = 24$

$\therefore$ Number of words begin with P and does not end with M = Number of words begin with P - Number of words begin with P and end with M

$= 24-6 = 18$

16. Solution:

We have,

$(1+3x)^6(1-x)^7 = [1 + {}^6C_1(3x) + {}^6C_2(3x)^2 + {}^6C_3(3x)^3 + {}^6C_4(3x)^4 + {}^6C_5(3x)^5 + {}^6C_6(3x)^6]$

$\times[1 - {}^7C_1(x) + {}^7C_2(x)^2 - {}^7C_3(x)^3 + {}^7C_4(x)^4 - {}^7C_5(x)^5 + {}^7C_6(x)^6 - {}^7C_7(x)^7]$

$=[1+7\times3x+15\times9x^2+20\times27x^3+15\times81x^4+6\times243x^5+1\times729x^6]$

$\times[1-7\times x+21\times x^2-35\times x^3+35\times x^4-21\times x^5+7\times x^6-1\times x^7]$

$=[1+21x+135x^2+540x^3+1215x^4+1458x^5+729x^6]$

$\times[1-7x+21x^2-35x^3+35x^4-21x^5+7x^6-x^7]$

$\therefore$ Co-efficient of the x^4 in the product

$=1\times35+21\times(-35)+135\times21+540\times(-7)+1215\times1$

$=35-735+2835-3780+1215=-430$

Hence, the co-efficient of the product of x^4 in the given expansion is −430.

17. Solution:

The equation of a line passing through the points (1,−3,6) and parallel to $x = y = z$ is

$\frac{x-1}{1}=\frac{y+3}{1}=\frac{z-6}{1}=\lambda$ (Let)

Thus, any point on this line is of the form (λ+1,λ−3,λ+6).

Now, if P(λ+1,λ−3,λ+6) is the point of intersection of line and plane, then

$\lambda+1-(\lambda-3)+\lambda+6=6$

$\therefore \lambda+1=-4+1=-3, \lambda-3=-4-3=-7, \lambda+6=-4+6=2$

∴Coordinates of point P are (−3,−7,2).

Hence, required distance

$= \sqrt{(-3-1)^2+(-7+3)^2+(2-6)^2}$

$=\sqrt{4^2+4^2+4^2}$

$=\sqrt{16+16+16}=\sqrt{48}=4\sqrt{3}$ units

18. Solution:

∵ The vertices of the ellipse lie on the $y-axis$, it is a vertical ellipse.

Let the required equation be $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$, where $a^2 > b^2$.

Its vertices are $(0, \pm a)$and therefore, $a = 5$.

Let $c^2=a^2-b^2$

Then, $e = \frac{c}{a} \Rightarrow c = ae = 5 \times \frac{3}{5} = 3$

Now, $c^2=a^2-b^2 \Leftrightarrow b^2=a^2-c^2=5^2-3^2=25-9=16$

$\therefore a^2=5^2=25$ and $b^2=16$

Hence, the required equation is $\frac{x^2}{16}+\frac{y^2}{25}=1$.

19. Solution:

$$L.H.S.=\begin{vmatrix} x+\lambda & 6x & 6x \\ 6x & x+\lambda & 6x \\ 6x & 6x & x+\lambda \end{vmatrix}$$

Applying $C_1 \to C_1+C_2+C_3$

$$L.H.S.=\begin{vmatrix} 13x+\lambda & 6x & 6x \\ 13x+\lambda & x+\lambda & 6x \\ 13x+\lambda & 6x & x+\lambda \end{vmatrix}$$

$$L.H.S.=(13x+\lambda)\begin{vmatrix} 1 & 6x & 6x \\ 1 & x+\lambda & 6x \\ 1 & 6x & x+\lambda \end{vmatrix}$$

Applying $R_2 \to R_2-R_1$, $R_3 \to R_3-R_1$

$$L.H.S.=(13x+\lambda)\begin{vmatrix} 1 & 6x & 6x \\ 0 & \lambda-5x & 0 \\ 0 & 0 & \lambda-5x \end{vmatrix}$$

$L.H.S.=(13x+\lambda)(\lambda-5x)^2=R.H.S.$

Hence, it is proved.

OR

Solution:

We have, $A=\begin{bmatrix} 2 & 6 \\ 6 & 8 \end{bmatrix}$

$$A^2=A\cdot A=\begin{bmatrix} 2 & 6 \\ 6 & 8 \end{bmatrix}\begin{bmatrix} 2 & 6 \\ 6 & 8 \end{bmatrix}$$

$$=\begin{bmatrix} 4+36 & 12+48 \\ 12+48 & 36+64 \end{bmatrix}=\begin{bmatrix} 40 & 60 \\ 60 & 100 \end{bmatrix}$$

$$\therefore A^2-20I_2=\begin{bmatrix} 40 & 60 \\ 60 & 100 \end{bmatrix}-20\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}=\begin{bmatrix} 40 & 60 \\ 60 & 100 \end{bmatrix}-\begin{bmatrix} 20 & 0 \\ 0 & 20 \end{bmatrix}$$

$$\therefore A^2-20I_2=\begin{bmatrix} 20 & 60 \\ 60 & 80 \end{bmatrix}=10\begin{bmatrix} 2 & 6 \\ 6 & 8 \end{bmatrix}=10A$$

$\Rightarrow A^2-20I_2=10A$

$\because kA=10A$

$\Rightarrow k=10$

Hence, $k=10$.

20. Solution:

Let $\cos^{-1}\frac{12}{13}=\theta$. Then, $\cos\theta=\frac{12}{13}$.

$\therefore\ \tan\theta=\frac{\sin\theta}{\cos\theta}=\frac{\sqrt{1-\cos^2\theta}}{\cos\theta}\ [\because \sin^2\theta+\cos^2\theta=1]$

$=\frac{\sqrt{1-\frac{144}{169}}}{\frac{12}{13}}=\frac{5}{13}\times\frac{13}{12}=\frac{5}{12}$

$\therefore \theta=\tan^{-1}\frac{5}{12}$

$L.H.S.=\tan^{-1}\frac{5}{12}+\tan^{-1}\frac{4}{3}$

$=\tan^{-1}\left(\frac{\frac{5}{12}+\frac{4}{3}}{1-\frac{5}{12}\cdot\frac{4}{3}}\right)\ \left(\because \tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-xy}\right)\right)$

$=\tan^{-1}\left(\frac{\frac{5+16}{12}}{\frac{36-20}{36}}\right)=\tan^{-1}\left(\frac{21}{12}\times\frac{36}{16}\right)=\tan^{-1}\left(\frac{7}{4}\times\frac{9}{4}\right)$

$=\tan^{-1}\left(\frac{63}{16}\right)=R.H.S.$

OR

Solution:

Given, $\tan^{-1}(x+3)+\tan^{-1}(x-3)=\tan^{-1}\frac{12}{3}, x>0$

$\Rightarrow\ \tan^{-1}\left(\frac{(x+3)+(x-3)}{1-(x+3)\cdot(x-3)}\right)=\tan^{-1}\frac{12}{3}$

$\left(\because \tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-xy}\right)\right)$

$+3+x-3 \Rightarrow \tan^{-1}\left(\frac{2x}{1-(x^2-9)}\right)=\tan^{-1}\frac{2}{3}$

$\Rightarrow \tan^{-1}\left(\frac{2x}{1-x^2+9}\right)=\tan^{-1}\frac{2}{3}$

$\Rightarrow \frac{2x}{10-x^2}=\frac{2}{3}$

$\Rightarrow 6x=20-2x^2$

$\Rightarrow 2x^2+6x-20=0$

$\Rightarrow x^2+3x-10=0$

$\Rightarrow x^2+5x-2x-10=0$

$\Rightarrow x(x+5)-2(x+5)=0$

$\Rightarrow (x+5)(x-2)=0$

$\Rightarrow x=2 \text{ or } -5$

But it is given that, $x>0$.

$\therefore x=2$.

21. Solution:

$1+21+41+61+\cdots+x=622500$

The given series is an AP

Here, first term (a)=1, Common difference $(d)=21-1=20$ and

x be the number of term

$S_x=622500$

Using formula,

$S_n=\frac{n}{2}\{2a+(n-1)d\}$

$S_x=\frac{x}{2}\{2\cdot 1+(x-1)20\}=622500$

$\Rightarrow \frac{x}{2}\{20x-18\}=622500$

$\Rightarrow 10x^2-9x-622500=0$

$\therefore\ x=\frac{-(-9)\pm\sqrt{(-9)^2-4\cdot 10\cdot(-622500)}}{2\cdot 10}=\frac{9\pm\sqrt{81+40\times 622500}}{20}$

$=\frac{9\pm\sqrt{24900081}}{20}=\frac{9\pm 4990}{20}$

$=\frac{9+4990}{20},\frac{9-4990}{20}$

$=249,-249.5$ (Negative sign neglected)

Hence, $x=249$.

OR

Solution:

We have, $x^2+3x+3=0$

Here, $a=1, b=3\ and\ c=3$

$\therefore D=b^2-4ac$

$=3^2-4\cdot 1\cdot 3$

$=9-12=-3<0$

So, the given equation has complex roots.

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$=\frac{-3\pm\sqrt{-3}}{2\cdot 1}$

$=\frac{-3\pm i\sqrt{3}}{2}\ [\because i=\sqrt{-1}]$

$\therefore$ Solution set $=\{\frac{-3+i\sqrt{3}}{2},\frac{-3-i\sqrt{3}}{2}\}$

22. Solution:

We have, $(m \vee n) \vee (\quad m \wedge n)$

$(m \vee n) \vee (\quad m \wedge n) \equiv (\quad m \wedge \quad n) \vee (\quad m \wedge n)$

[∵De-Morgan's law $(x \vee y) = (\quad x \wedge \quad y)$]

$\equiv \quad m \wedge (\quad m \vee n)$ [∵ Distributive law $x \vee y = t)$]

$\equiv \quad m \wedge t$

$\equiv \quad m$

23. Solution:

Let the probability that Raju can solve a problem be denoted by P(R).

So, $P(R) = \frac{1}{3}$

Let the probability that Akash can solve a problem be denoted by P(A).

So, $P(A) = \frac{3}{4}$

Also, $P(X \cap Y) = P(X) \cdot P(Y)$

Hence,

$P(R \cap A) = P(R) \cdot P(A) = \frac{1}{3} \times \frac{3}{4} = \frac{1}{4}$

Also, $P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$

$P(R \cup A)$ represents that both of them will solve the problem.

$\therefore P(R \cup A) = P(R) + P(A) - P(R \cap A)$

$P(R \cup A) = \frac{1}{3} + \frac{3}{4} - \frac{1}{4}$

$= \frac{4+9-3}{12} = \frac{10}{12} = \frac{5}{6}$

Hence, required answer is $\frac{5}{6}$

24. Solution:

We have, $f(x) = |\log 11 - \sin x|$ and $g(x) = f(f(x))$, $x \in R$

Note that $x \to 0, \log 11 > \sin x$

$$\therefore f(x) = \log 11 - \sin x$$

$\Rightarrow g(x) = \log 11 - \sin f(x)) = \log 11 - \sin(\log 11 - \sin x)$

Clearly, $g(x)$ is differentiable at x=0 as $\sin x$ is differentiable.

Now,
$g'(x) = 0 - \cos(\log 11 - \sin x)(-\cos x)$
$= \cos x . \cos(\log 11 - \sin x)$

$\Rightarrow g'(0) = \cos 0 \cdot \cos(\log 11 - \sin 0) = 1 \cdot \cos(\log 11)$

Hence, $g'(0) = \cos(\log 11)$.

OR

Solution:

Given,

$$f(x) = \begin{cases} (1 + |\sin\theta|)^{\frac{a}{|\sin\theta|}}, & -\frac{\pi}{6} < \theta < 0 \\ b, & \theta = 0 \\ e^{\tan 7\theta / \tan 8\theta}, & 0 < \theta < \frac{\pi}{6} \end{cases}$$

Since, $f(x)$ is continuous at $x = 0$, therefore

$\lim_{x \to 0^-} f(x) = f(0) = \lim_{x \to 0^+} f(x)$

$\Rightarrow \lim_{x \to 0^-} (1 + |\sin\theta|)^{\frac{a}{|\sin\theta|}} = b = \lim_{x \to 0^+} e^{\tan 7\theta / \tan 8\theta}$

$\Rightarrow e^a = b = e^{\frac{7}{8}}$

$\Rightarrow a = \frac{7}{8}$ and $a = \log_e b$

Hence, $a = \frac{7}{8}$ and $a = \log_e b$.

25. Solution:

Let a be the first term and r be the common ratio.

$a + ar + ar^2 + \cdots + \infty = 8$

$\Rightarrow \frac{a}{1-r} = 8$

Squaring both sides, we get

$\frac{a^2}{(1-r)^2} = 64$

$\Rightarrow a^2 = 64(1-r)^2 \quad .....(1)$

Also,

$a^2 + a^2r^2 + a^2r^4 + \cdots + \infty = 4$

$\frac{a^2}{1-r^2} = 4 \quad .....(2)$

Putting the value of (1) in (2), we get

$\frac{64(1-r)^2}{1-r^2} = 4$

$\Rightarrow \frac{16(1-r)^2}{(1+r)(1-r)} = 1$

$\Rightarrow \frac{16(1-r)}{(1+r)} = 1$

$\Rightarrow 16 - 16r = 1 + r \Rightarrow 17r = 15$

$\therefore r = \frac{15}{17}$

Putting $r = \frac{15}{17}$ in $\frac{a}{1-r} = 8$, we get

$\frac{a}{1-\frac{15}{17}} = 8 \Rightarrow a = \frac{16}{17}$

Hence, first term $(a) = \frac{16}{17}$ and common difference $(r) = \frac{15}{17}$.

26. Solution:

Given matrix is $A=\begin{bmatrix} -3 & 3 & 6 \\ 3 & 6 & 9 \\ 9 & 3 & 3 \end{bmatrix}$

Let $A = IA$

$$\Rightarrow \begin{bmatrix} -3 & 3 & 6 \\ 3 & 6 & 9 \\ 9 & 3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \rightarrow R_2 + R_1, R_3 \rightarrow R_3 + 3R_1$, we get

$$\Rightarrow \begin{bmatrix} -3 & 3 & 6 \\ 0 & 9 & 15 \\ 0 & 12 & 21 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \rightarrow (-1)R_1$, we get

$$\Rightarrow \begin{bmatrix} 3 & -3 & -6 \\ 0 & 9 & 15 \\ 0 & 12 & 21 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 1 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \rightarrow R_2 - R_3$, we get

$$\Rightarrow \begin{bmatrix} 3 & -3 & -6 \\ 0 & -3 & -6 \\ 0 & 12 & 21 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ -2 & 1 & -1 \\ 3 & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \rightarrow R_1 - R_2, R_3 \rightarrow R_3 + 4R_2$, we get

$$\Rightarrow \begin{bmatrix} 3 & 0 & 0 \\ 0 & -3 & -6 \\ 0 & 0 & -3 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ -2 & 1 & -1 \\ -5 & 4 & -3 \end{bmatrix} A$$

Applying $R_2 \rightarrow (-1)R_2$, we get

$$\Rightarrow \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 6 \\ 0 & 0 & -3 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 1 \\ -5 & 4 & -3 \end{bmatrix} A$$

Applying $R_2 \rightarrow R_2 + 2R_3$, we get

$$\Rightarrow \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -3 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ -5 & 4 & -3 \end{bmatrix} A$$

Applying $R_3 \rightarrow (-1)R_3$, we get

$$\Rightarrow \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ 5 & -4 & 3 \end{bmatrix} A$$

$$\Rightarrow (27)\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ 5 & -4 & 3 \end{bmatrix} A$$

$$\Rightarrow A^{-1} = \frac{1}{27}\begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ 5 & -4 & 3 \end{bmatrix}$$

27. Solution:

We have, $(\tan x)^y = (\tan y)^x$,

On taking log both sides, we get

$y \log(\tan x) = x \log(\tan y)$ $[\because \log m^n = n \log m]$ ... (1)

On differentiating both sides of (1) w.r.t. x, we get

$$y \cdot \frac{d}{dx}(\log(\tan x)) + \log(\tan x)\frac{d}{dx}(y) = x \cdot \frac{d}{dx}(\log(\tan y)) + \log(\tan y)\frac{d}{dx}(x)$$

$$\Rightarrow y \cdot \frac{1}{(\tan x)} \cdot \frac{d}{dx}(\tan x) + \log(\tan x)\frac{dy}{dx} = x \cdot \frac{1}{(\tan y)} \cdot \frac{d}{dx}((\tan y)) + \log(\tan y) \cdot 1$$

$$\Rightarrow y \cdot \frac{1}{(\tan x)} \cdot \sec^2 x + \log(\tan x)\frac{dy}{dx} = x \cdot \frac{1}{(\tan y)} \cdot \sec^2 x \cdot \frac{dy}{dx} + \log(\tan y)$$

$$\Rightarrow \log(\tan x)\frac{dy}{dx} - x \cdot \frac{1}{(\tan y)} \cdot \sec^2 x \cdot \frac{dy}{dx} = \log(\tan y) - y \cdot \frac{1}{(\tan x)} \cdot \sec^2 x$$

$$\Rightarrow \left(\log(\tan x) - x \cdot \frac{1}{(\tan y)} \cdot \sec^2 x\right)\frac{dy}{dx} = \frac{(\tan x) \cdot \log(\tan y) - y \cdot \sec^2 x}{(\tan x)}$$

$$\Rightarrow \left(\frac{(\tan y) \cdot \log(\tan x) - x \cdot \sec^2 x}{(\tan y)}\right)\frac{dy}{dx} = \frac{(\tan x) \cdot \log(\tan y) - y \cdot \sec^2 x}{(\tan x)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{(\tan y)((\tan x) \cdot \log(\tan y) - y \cdot \sec^2 x)}{(\tan x)((\tan y) \cdot \log(\tan x) - x \cdot \sec^2 x)}$$

Hence, it is proved.

OR

Solution:

Let $P(n)$: $51n-14n$, $\forall\ n \in N$

For n=1, the given expression becomes

$511-141=51-14=37$, which is multiple of 37.

So, the given statement is true for 1,*i.e.*, $P(1)$ is true.

Let $P(k)$ be true.

Then, $P(k)$: $51k-14k$ is multiple of 37.

$\Rightarrow 51k-14k=37m$ for some natural number m. ...(1)

Now, $51k+1-14k+1=(51k+1-51\cdot14k)+(51\cdot14k-14k+1)$

[On subtracting and adding $51\cdot14k$]

$=51(51k-14k)+14k(51-14)$

$=51\cdot37m+37\cdot14k$ [Using(1)]

$=37(51m+14k)$, Which is multiple of 37.

$\therefore$ $P(k+1)$: $51k+1-14k+1$ is multiple of 37.

$\Rightarrow P(k+1)$ is true, whenever $P(k)$ be true.

Thus, $P(1)$ is true and $P(k+1)$ is true, whenever $P(k)$ be true.

Hence, by principle of mathematical induction, $P(n)$ is true for all $n \in N$.

28. Solution:

The given function is

$f(x)=x3-2x+9$

On differentiating both sides $w.r.t.x$, we get

$f'(x) = 3x - 2$

On putting $f'(x)=0$, we get

$\Rightarrow 3x-2=0$

$\Rightarrow 3x = 2$

$\Rightarrow x = \frac{2}{3}$

Now, we find intervals in which $f(x)$ is strictly increasing or strictly decreasing.

Sign of $f'(x)$	Interval	$f'(x) = 3x - 2$	
$-ve$	$x < 1$	$(-)$	
$+ve$	$x > 1$	$(+)$	

We know that, a function $f(x)$ is said to be strictly increasing, if $f'(x)>0$ and it is said to be strictly increasing, if $f'(x)<0$. So, the given function $f(x)$ is:

1. Strictly increasing on the interval $(\frac{2}{3}, \infty)$.

2. Strictly decreasing on the interval $(-\infty, \frac{2}{3})$.

Hence, $f(x)$ increasing nor decreasing in $(-1,1)$.

29. Solution:

Given curves are $y = \sqrt{x}$ ......(1)

and $y - x + 2 = 0$ ......(2)

On solving (1) and (2), we get

$\sqrt{x} - \sqrt{x}^2 + 2 = 0$

$\Rightarrow \sqrt{x}^2 - \sqrt{x} - 2 = 0$

$\Rightarrow \sqrt{x}^2 - 2\sqrt{x} + \sqrt{x} - 2 = 0$

$\Rightarrow \sqrt{x}(\sqrt{x} - 2) + 1(\sqrt{x} - 2) = 0$

$\Rightarrow (\sqrt{x} - 2)(\sqrt{x} + 1) = 0$

$\Rightarrow \sqrt{x} = 2$ [$\because \sqrt{x} = -1$ is not possible]

Hence, required area $= \int_0^2 (x_2 - x_1)\, dy = \int_0^2 (y+2)\, dy - \int_0^2 y^2\, dy$

$= \left[\frac{y^2}{2} + 2y - \frac{y^3}{3}\right]_0^2$

$= 2 + 4 - \frac{8}{3} = \frac{10}{3}$ sq. units

OR

Solution:

Given equations of lines are:

$\frac{x-1}{2} = \frac{y-3}{\lambda} = \frac{z+1}{-1}$ and $\frac{x+1}{\lambda} = \frac{y-1}{2} = \frac{z-2}{2}$.

The given lines are parallel to the vectors $\vec{b}_1 = 2\hat{i} + \lambda\hat{j} - \hat{k}$ and $\vec{b}_2 = \lambda\hat{i} + 2\hat{j} + 2\hat{k}$ respectively. The lines are perpendicular if $\vec{b}_1 \cdot \vec{b}_2 = 0$

$\Rightarrow (2\hat{i} + \lambda\hat{j} - \hat{k})(\lambda\hat{i} + 2\hat{j} + 2\hat{k}) = 0$

$\Rightarrow 2 \times \lambda + \lambda \times 2 + (-1) \times 2 = 0$

$\Rightarrow 4\lambda - 2 = 0$

$\Rightarrow \lambda = \frac{2}{4}$

Hence, $\lambda = \frac{1}{2}$.